H1 Terminologie en Eigenschappen van Fluïda

Eigenschappen

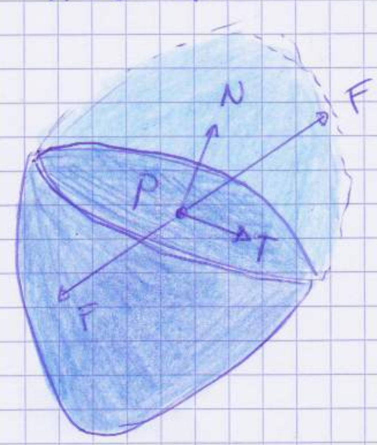
Druk p - Schuifspanning τ

De druk in een fluïdum in rust is dus gedefinieerd als de normaalkracht per eenheidsoppervlakte uitgeoefend op een imaginair vlak in het fluïdum.

$$p = \lim_{A \to 0} \left(-\frac{N}{A} \right) \to \text{normaalspanning}$$

Een fluïdum is een stof die niet in statisch evenwicht blijft onder invloed van schuifspanningen. Een fluïdum zal vervormen zolang er een schuifkracht op uitgeoefend is, hoe klein ook.

$$\tau = \lim_{A \to 0} \left(\frac{T}{A} \right) \to \text{tangentiële spanning}$$



Samendrukbaarheid

volumetrische elasticiteits modulus

$$E_v = \frac{\text{Drukverandering}}{\text{Relatieve volumeverandering}} = -\frac{dp}{\left(\frac{dV}{V}\right)}$$

druktoename
+
volume vermindering

$m = ct. \rightarrow \rho V = ct \rightarrow \rho dV + d\rho V = 0$

$$\frac{dV}{V} = -\frac{d\rho}{\rho}$$

$$\Rightarrow \quad E_v = \frac{dp}{\left(\frac{d\rho}{\rho}\right)} \quad \rightsquigarrow \quad \chi \text{ compressie coëff.} = \frac{1}{E_v}$$

Drukverandering

$$dV = -\chi V dp$$

$$\int_{\rho_0}^{\rho} \frac{d\rho}{\rho} = \int_{P_0}^{P} \chi \, dP \quad \rightsquigarrow \quad \ln \frac{\rho}{\rho_0} = \chi (P - P_0)$$

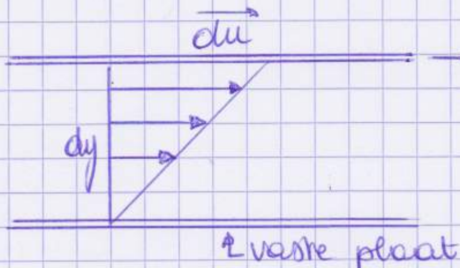
Temp. verandering

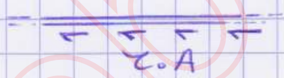
$$dV = \alpha V dT$$

$$-\frac{d\rho}{\rho} = \alpha \, dT \quad \rightsquigarrow \quad \ln \frac{\rho}{\rho_0} = -\alpha (T - T_0)$$

Viscositeit

De viscositeit is een maat voor de 'stroperigheid', voor deze weerstand v/h fluïdum tegen schuifspanningen.





(dyn.) viscositeit

$$\tau = \mu \frac{du}{dy}$$

snelheidsgradiënt

Enkel voor Newtoniaanse vloeistoffen

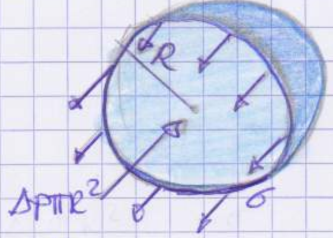
kinematische viscositeit: $\nu\ [m^2/s] = \frac{\mu}{\rho}$

Oppervlaktespanning, capillariteit

De opp. spanning σ is een maat voor de attractiekracht op de opp. moleculen en is gedef. als de arbeid per eenheidsopp. nodig om het opp. te vergroten.

bolvormige druppel:

$$F = \Delta P \cdot A = \Delta P \pi R^2$$



$$F = \sigma \cdot \text{omtrek} \quad (\text{rand v/d opp. druppel}) = \sigma \cdot 2\pi R$$

$$\Rightarrow \quad \Delta P = \frac{2\sigma}{R}$$

de druk neemt toe met afnemende druppeldiameter

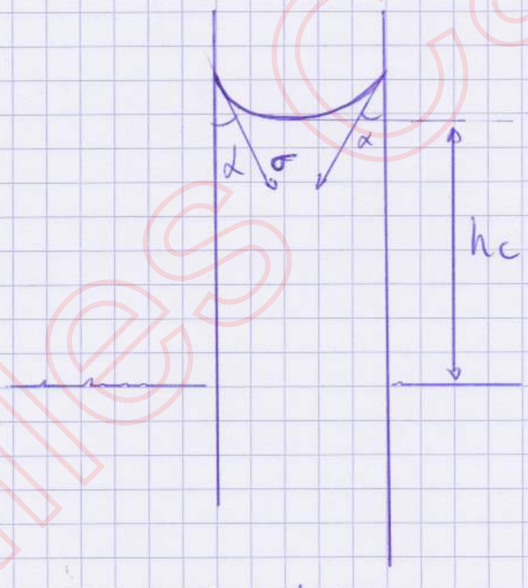
De stijghoogte h_c :

verticale component v/d resultante v/d opp. spanning = gewicht v/d vloeistofkolom

$$2\pi r \sigma \cdot \cos\alpha = \pi r^2 \cdot \rho g h_c$$

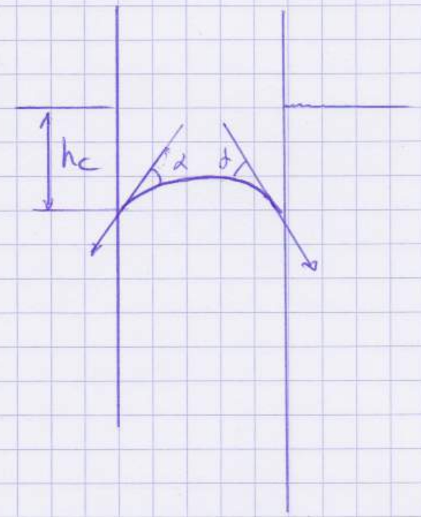
$$\Rightarrow \quad h_c = \frac{2\sigma \cos\alpha}{\rho g \cdot r}$$

adh > coh.



hydrofiele wand

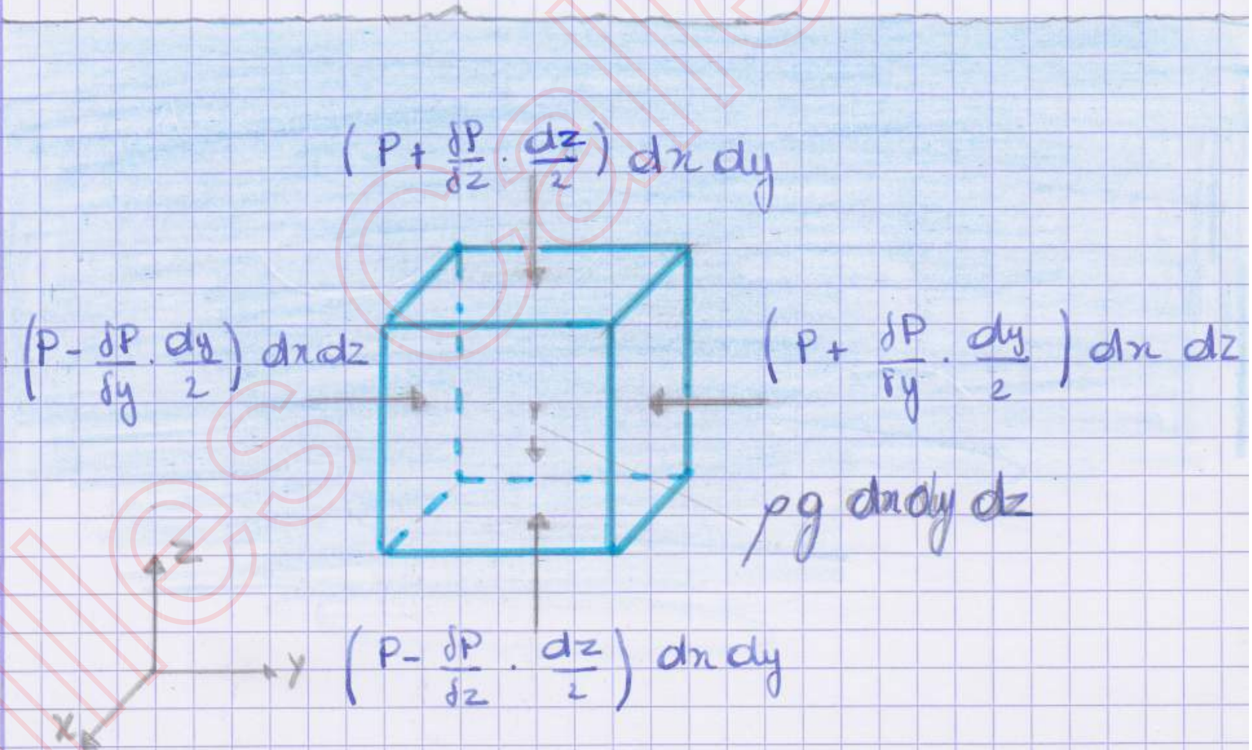
adh < coh.



hydrofobe wand

H2 Statica der Fluïda

Basisverg. voor het verloop v/d druk in fluïda



In rust:

$$\delta F_y = \left(P - \frac{\delta P}{\delta y} \cdot \frac{dy}{2}\right) dx\, dz - \left(P + \frac{\delta P}{\delta y} \cdot \frac{dy}{2}\right) dx\, dz$$

$$0 = \frac{\delta P}{\delta y} \underbrace{dy\, dx\, dz}_{\neq 0} \Rightarrow \boxed{\frac{\delta P}{\delta y} = 0}$$

analoog

$$\boxed{\frac{\delta P}{\delta x} = 0}$$

$$\delta F_z = -\frac{\delta P}{\delta z} dx\, dy\, dz - \rho g\, dx\, dy\, dz$$

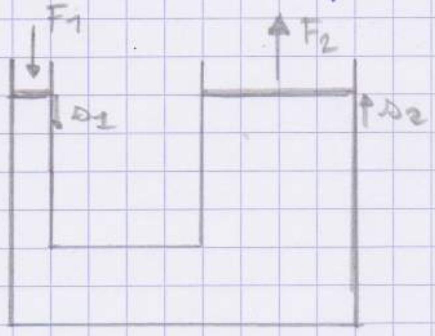
$$\boxed{\frac{\delta P}{\delta z} = -\rho g}$$

Onsamendrukbare fluïda

Hydrostatische wet: $\Delta P = -\rho g \Delta z$

$$P_1 = P_2 + \rho g h$$

Hydraulische pers:



de druk op beide zuigers is gelijk

$$p = \frac{F_1}{A_1} = \frac{F_2}{A_2}$$

$$\Rightarrow F_2 = \frac{F_1}{A_1} \cdot A_2$$

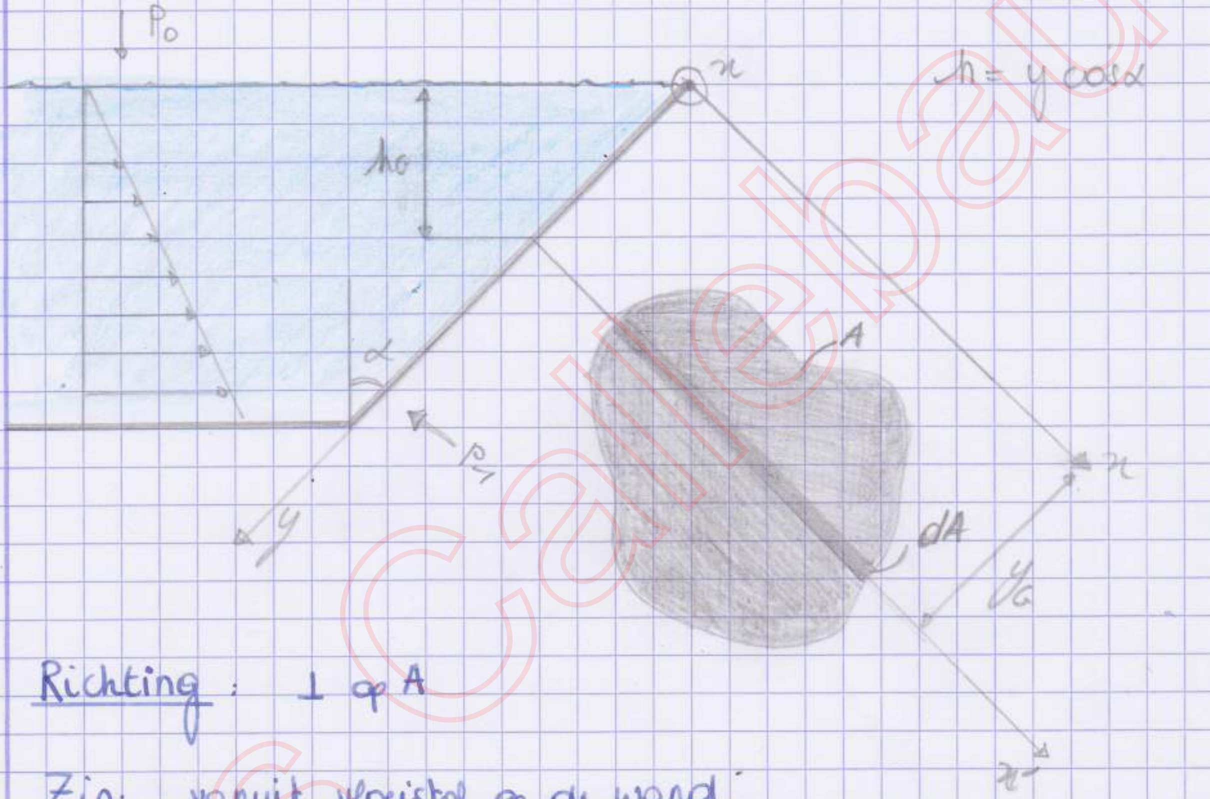
$$\Delta V_1 = \Delta V_2$$

$$\Rightarrow s_2 = s_1 \frac{A_1}{A_2}$$

Drukmeting

lees p 2-8 t.e.m p 2-13

Hydrostatische krachten op een vlakke wand



<u>Richting</u>: $\perp$ op A

<u>Zin:</u> vanuit vloeistof op de wand

<u>Grootte:</u> $\Delta P = (P_0 + \rho g h) - P_1$

$$dF = \Delta P . dA = \left[(P_0 + \rho g h) - P_1 \right] dA$$

$$\downarrow$$

$$F_{tot} = \int_A (P_0 - P_1) + \rho g h \, dA$$

$$= \int_A (P_0 - P_1)\, dA + \int_A \rho g h \, dA$$

$$= (P_0 - P_1) . A + \rho g \int_A h_G \, dA$$

$$F_{tot} = \underbrace{(P_0 - P_1) . A}_{\text{aangrijpingspunt in } (x_G, y_G)} + \underbrace{\rho g \cos\alpha \, y_G A}_{\text{aangrijpingspunt in } (x_M, y_M)}$$

$x_M \rightarrow M_{oy}$

dA

$$dF = \rho g h \, dA$$

$$dM_{oy} = x \, dF = (\rho g h \, dA) \, x$$

A

$$M_{oy} = \int_A dM_{oy} = \int_A \rho g h x \, dA$$

$$M_{oy} = F_G \cdot x_M = x_M \, \rho g h_G A$$

$$\Rightarrow x_M = \frac{M_{oy}}{\rho g h_G A}$$

$$x_M = \frac{\rho g \int_A h \cdot x \cdot dA}{\rho g \, h_G \cdot A}$$

$h = y \cos\alpha$

$h_G = y_G \cos\alpha$

$$x_M = \frac{\int_A xy \, dA}{y_G \cdot A}$$

$$\int_A xy \, dA = I_{xy}$$

$$x_M = \frac{I_{xy}}{y_G \cdot A}$$

$y_M \rightarrow M_{ox}$

$$M_{ox} = \int_A dM_{ox} = \int_A \rho g h y \, dA$$

$$M_{ox} = y_M F_G \Rightarrow y_M = \frac{M_{ox}}{\rho g h_G A}$$

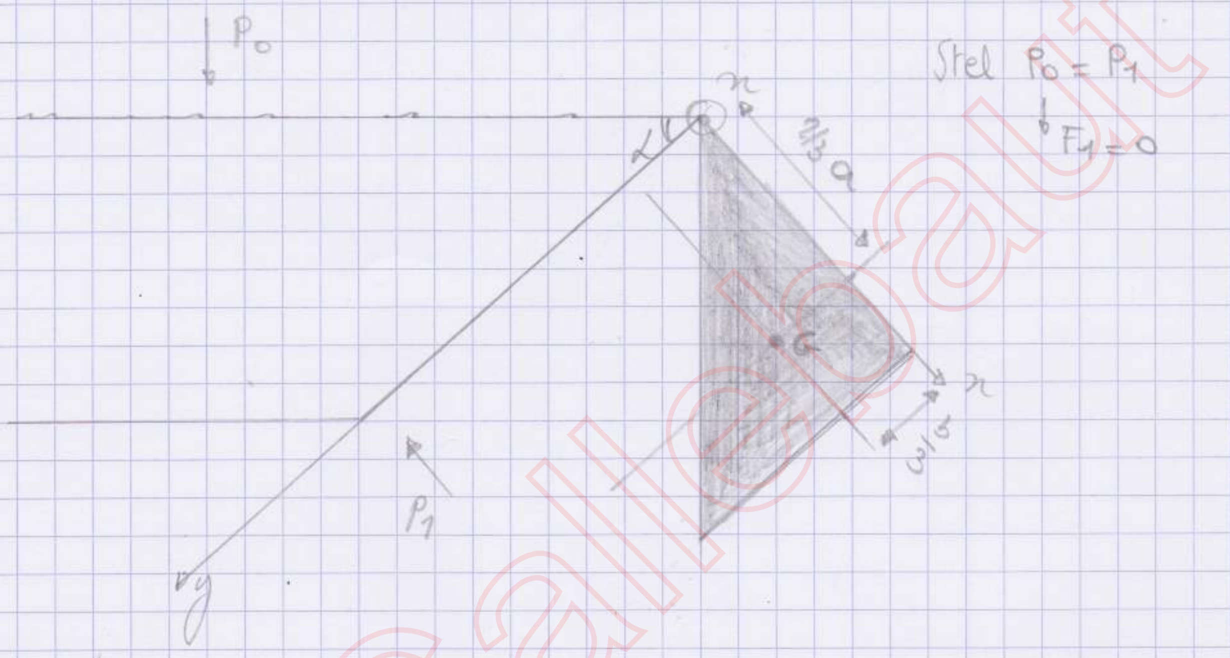
$$y_M = \frac{\int_A y^2 dA}{y_G A} = \frac{I_{xx}}{y_G \cdot A}$$

steiner :

$$I_{xx} = I_{x'x'} + y_G^2 A$$

$$y_M = \frac{I_{x'x'}}{y_G A} + y_G$$

Krachten op driehoekig stuk



$$F_{tot} = F_g = \rho g h_G \cdot A = \rho g \left(\frac{b}{3} \sin\alpha \right) \frac{ab}{2}$$

$$F_{tot} = \rho g \frac{ab^2}{6} \sin\alpha$$

$$x_D = \frac{I_{xy}}{y_G A}$$

$$I_{xy} = \int_A xy \, dA$$

$$= \int_0^a \int_0^{\frac{b}{a}x} xy \, dy \, dx$$

$$= \frac{a^2 b^2}{8}$$

$$x_D = \frac{a^2 b^2}{8} \cdot \frac{1}{\frac{b}{3} \frac{ab}{2}}$$

$$\hookrightarrow x_D = \frac{3}{4} a$$

$$y_D = \frac{I_{x'x'}}{y_G \cdot A} + y_G$$

gegeven: $\frac{ab^3}{36}$

als $I_{x'x'}$ niet gegeven:

$$I_{xx} = \int_A y^2 \, dA$$

$$= \int_0^a \int_0^{b/a\, x} y^2 \, dy \, dx$$

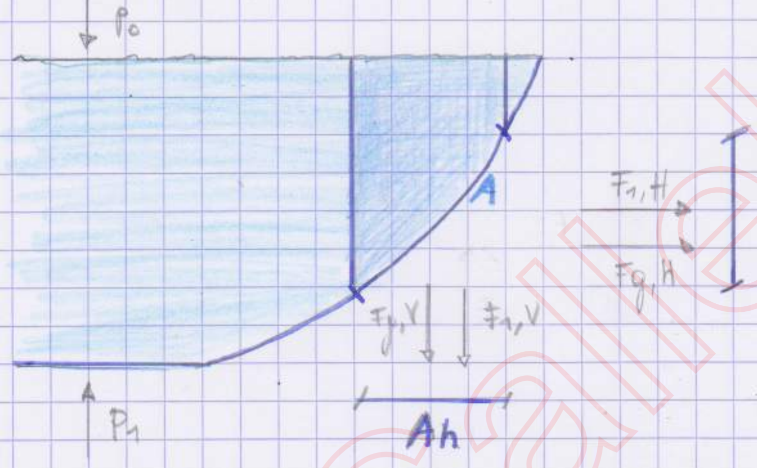
$$= \frac{ab^3}{12}$$

$$\hookrightarrow y_D = \frac{I_{xx}}{y_G \cdot A}$$

$$\hookrightarrow y_D = \frac{b}{2}$$

Hydrostatische krachten op een gekromd oppervlak

Vloeistof boven opp.



Op A_V

$F_{1,H} = (p_0 - p_1) . A_V$

$F_{g,H} = \rho g \underbrace{h_G}_{G \text{ van } A_V} A_V$

+

$F_H = [(p_0 - p_1) + \rho g h_G] A_V$

Op A_H

$F_{1,V} = (p_0 - p_1) A_H$

$F_{g,V} = G = mg = \rho V g$

+

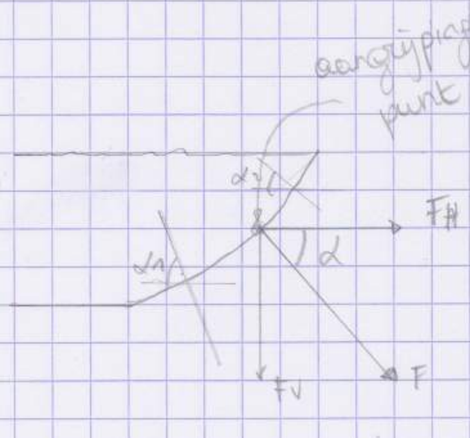
$F_V = A_H (p_0 - p_1) + \rho V g$

$\Downarrow$

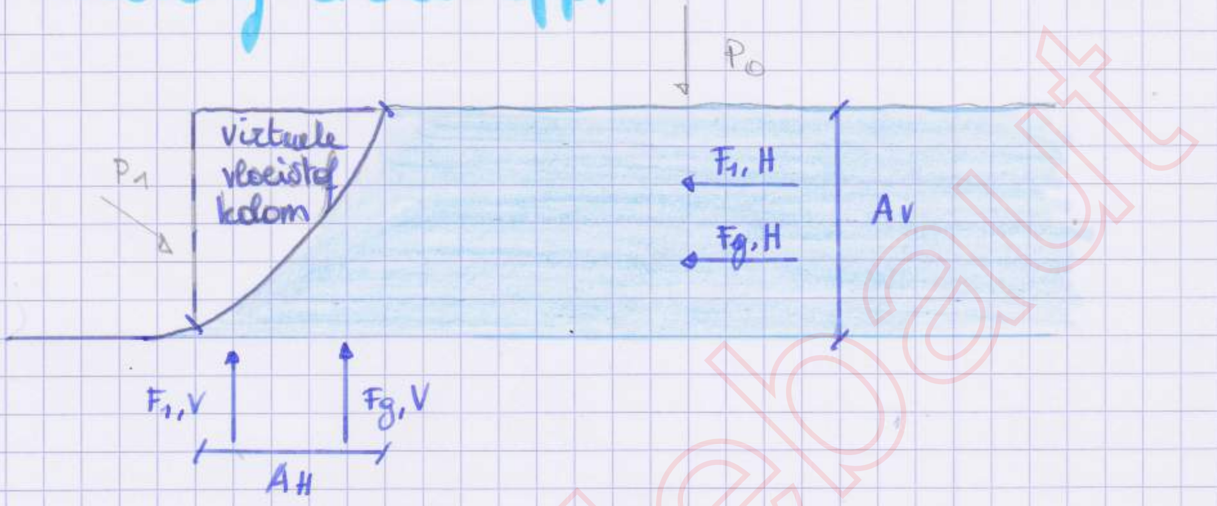
Op A

$F_{tot} = \sqrt{F_H^2 + F_V^2}$

$tg\alpha = \frac{F_V}{F_H}$



vloeistof onder opp.



<u>Op A_V</u>

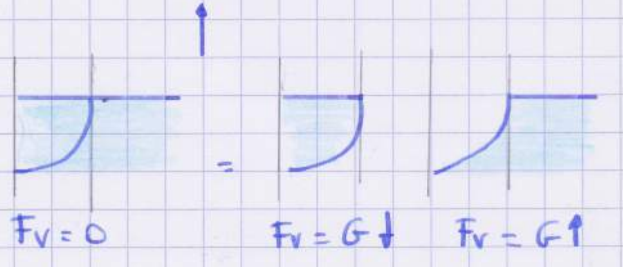
$F_{1,H} = (P_0 - P_1) A_V$

$F_{g,H} = \rho g h_G A_V$

<u>Op A_H</u>

$F_{1,V} = (P_0 - P_1) A_H$

$F_{g,V} = G_{uit} = \rho V_{uit} g$



$\Downarrow$

<u>Op A</u>

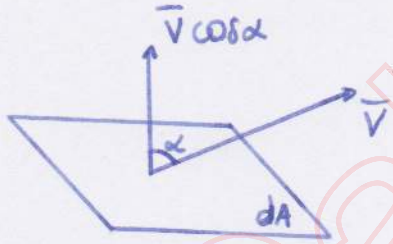
$$F = \sqrt{F_H^2 + F_V^2}$$

$$tg\,\alpha = \frac{F_V}{F_H}$$

H3 Beweging van volmaakte fluïda

Definities

Debiet $\quad Q = dA \cdot v \cdot \cos\alpha \qquad Q = \int_A v(A)\, dA = v_{\perp gem} \cdot A_{tot}$



$\dot{M} = \rho\, Q_{tot}\ [kg/s] \qquad \dfrac{\dot{M}}{A_{tot}} = \rho\, v_{gem}\ [kg/m^2 s]$

Permanente en nt permanete beweging

↳ stationair $\qquad$ ↳ uniforme

$\neq f(t) \qquad = f(t)$

Eenparige en nt eenparige stroming

nt plaatsafh. $\qquad$ plaatsafh.

Stroomlijn

een lijn die in elk punt raakt ad. ogenblikkelijke snelheid ve deeltjes dat zich op deze lijn bevindt.

snelheidsvector → rakend

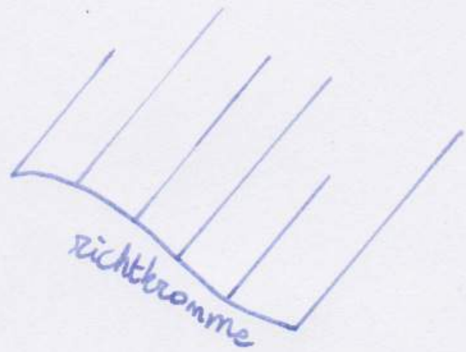
$\overline{V} = V\overline{e}_s$ met $V = \frac{ds}{dt}$

$\vec{V}_\perp = 0$

Stroomopp. en stroombuis

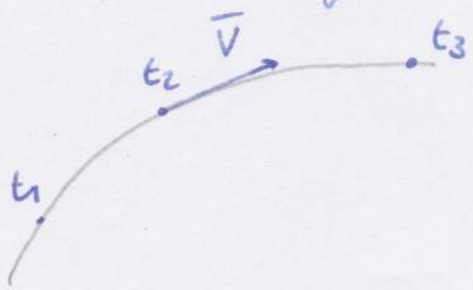
↓ in elk punt ve richtkromme een stroomlijn te teken

↓ stroomopp. met een gesloten richtkromme



Stroombaan

↳ de meetkundige plaats vd opeenvolgende posities van één deeltje in de tijd.



⇒ permanente stroming: stroomlijn = stroombaan.

Continuïteitsverg.

Wet van behoud van massa

$$\frac{dM_{sys}}{dt}=0 \quad \xRightarrow{dV=ct.} \quad \frac{m_{in}}{dt}-\frac{m_{uit}}{dt}=\frac{\Delta m}{dt}$$

$$\Big\Downarrow \text{ stationair}$$

$$\dot m_{in}=\dot m_{uit}$$

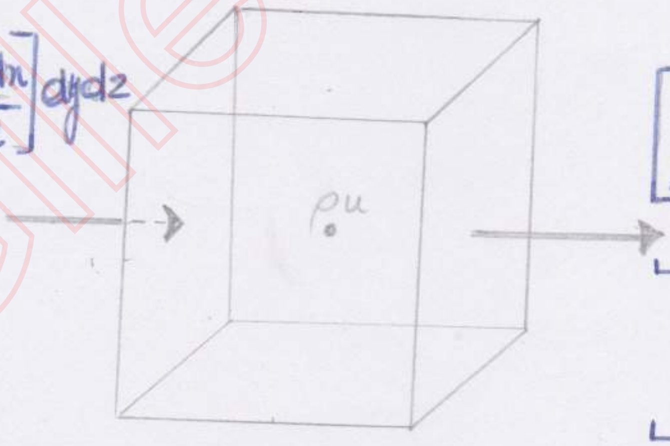
Differentiaalvolume:

$$dV = dx\,dy\,dz \qquad \vec v\,(u,v,w)$$

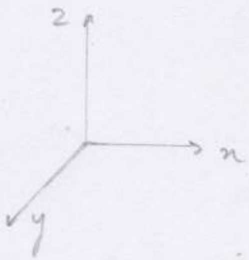
$$\dot M = \rho Q$$

$$\frac{\dot M}{A}=\rho v$$

$$\left[\rho u-\frac{\delta(\rho u)}{\delta x}\frac{dx}{2}\right]dydz$$



$$\underbrace{\underbrace{\left[\rho u+\frac{\delta(\rho u)}{\delta x}\frac{dx}{2}\right]}_{\frac{\dot M_{uit,x}}{A}}dydz}_{\dot M_{uit,x}}$$



in x-richting:

$$\left[\rho u - \frac{\delta(\rho u)}{\delta x}\frac{dx}{2}\right] dy\,dz\,dt - \left[\rho u + \frac{\delta(\rho u)}{\delta x}\frac{dx}{2}\right] dy\,dz\,dt$$

$$= -\frac{\delta(\rho u)}{\delta x}\, dV\,dt$$

in y-richting:

$$= -\frac{\delta(\rho v)}{\delta y}\, dV\,dt$$

in z-richting:

$$= -\frac{\delta(\rho w)}{\delta z}\, dV\,dt$$

totale netto-ingestroomde massa in controle volume:

$$-\left[\frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z}\right] dV\,dt \qquad (1)$$

massaverandering in tijd dt:

$$\underbrace{\left[\rho + \frac{\delta\rho}{\delta t}dt\right]}_{\rho(t+dt)} dV - \underbrace{\rho}_{\rho(t)}\, dV = \frac{\delta\rho}{\delta t}\, dt\, dV \qquad (2)$$

(1) = (2)

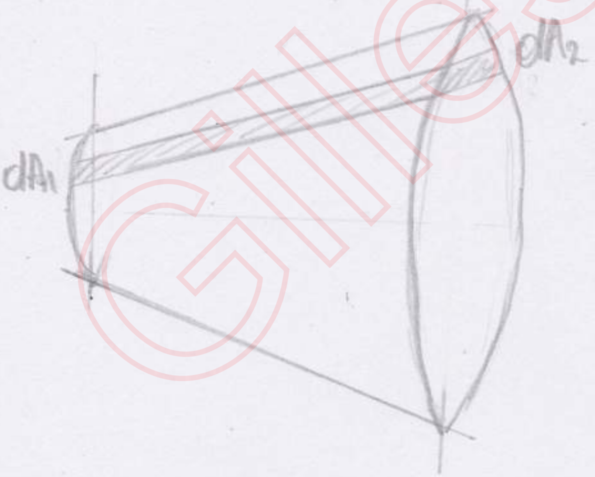
$$\frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z} + \frac{\delta \rho}{\delta t} = 0$$

$$\xRightarrow{\text{permanent}} \quad \frac{\delta(\rho u)}{\delta x} + \frac{\delta(\rho v)}{\delta y} + \frac{\delta(\rho w)}{\delta z} = 0$$

$$\underset{\text{drukbaar}}{\xRightarrow{\text{onsamen-}}} \quad \frac{\delta u}{\delta x} + \frac{\delta v}{\delta y} + \frac{\delta w}{\delta z} = 0$$

Wet van Castelli

wanden → stroomlijnen → gn stroming door de wand



$$d\dot{M}_{in} = d\dot{M}_{uit}$$

$$\rho\, d\dot{Q}_{in} = \rho\, d\dot{Q}_{uit}$$

$$d\dot{Q}_{in} = d\dot{Q}_{uit}$$

$$\underbrace{v_1}_{\text{gemiddelde snelheid}} dA_1 = \underbrace{v_2}_{} dA_2$$

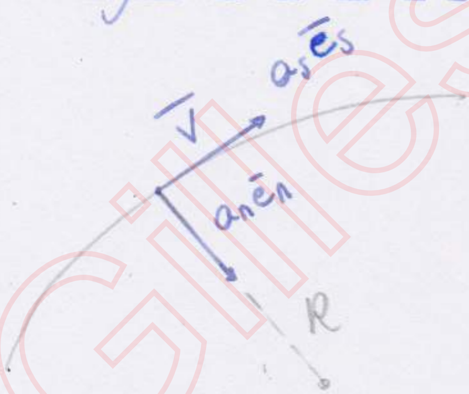
Kinematica v.e. fluïdumdeeltje

Versnelling v.e. fluïdumdeeltje

Cartesiaanse coördinaten

$$\bar{a} = \frac{d\bar{V}}{dt} = \underbrace{\frac{\delta \bar{V}}{\delta t}}_{\text{lokale versnelling}} + \underbrace{\left(\frac{\delta \bar{V}}{\delta x} \overset{u}{\widetilde{\frac{dx}{dt}}} + \frac{\delta \bar{V}}{\delta y} \overset{v}{\widetilde{\frac{dy}{dt}}} + \frac{\delta \bar{V}}{\delta z} \overset{w}{\widetilde{\frac{dz}{dt}}} \right)}_{\text{convectieve / relatieve versnelling}}$$

Stroomlijncoördinaten



$$\bar{a} = \frac{d\bar{V}}{dt} = \frac{\delta \bar{V}}{\delta t} + \frac{\delta \bar{V}}{\delta s} \overset{V}{\frac{ds}{dt}}$$

met $\bar{V} = V \bar{e}_s$

$$\Rightarrow \bar{a} = a_s \bar{e}_s + a_n \bar{e}_n$$

$$\frac{\delta \bar{e}_s}{\delta s} = \frac{\bar{e}_n}{R}$$

$$= V \left(\frac{\delta V}{\delta s} \bar{e}_s + V \frac{\delta \bar{e}_s}{\delta s} \right)$$

$$\begin{cases} a_s = V \dfrac{\delta V}{\delta s} = \dfrac{1}{2} \dfrac{\delta (V^2)}{\delta s} = \dfrac{1}{2} \dfrac{d(V^2)}{ds} \\[2em] a_n = \dfrac{V^2}{R} \end{cases}$$

Tweede wet van Newton

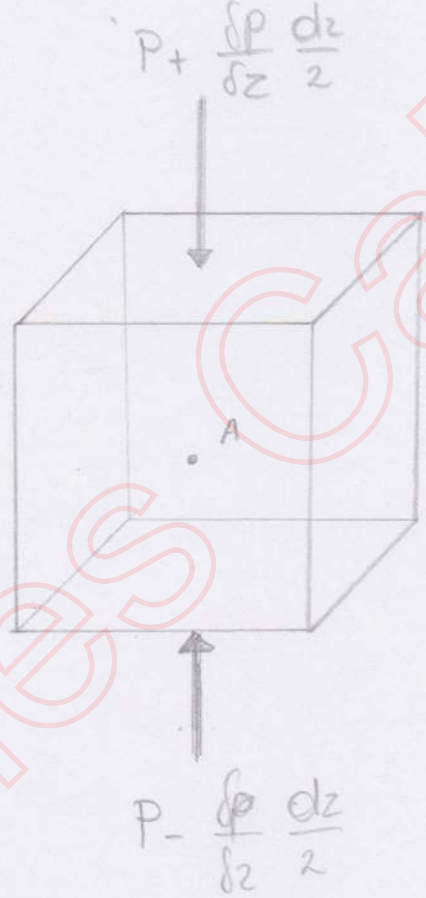
Algemene bewegingsverg voor ideale fluïda

$$\sum \overline{F} = (\rho\, dx\, dy\, dz)\, \overline{a}$$

zonder schuifspanningen

uitwendige krachten:

$$\overline{f}_u = \frac{\overline{F}_u}{dx\,dy\,dz}$$

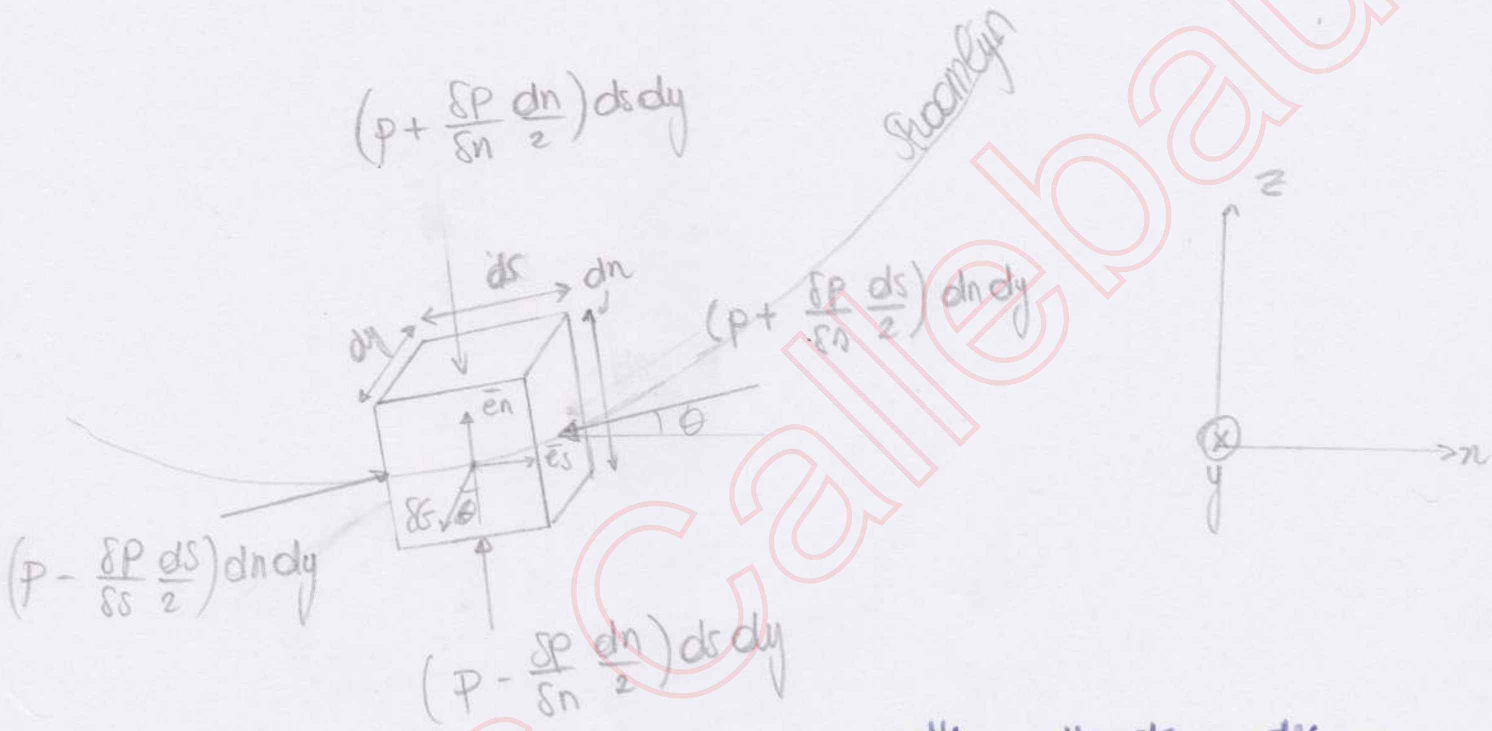


$$\begin{array}{l} x \\ \\ y \\ \\ z \end{array} \begin{cases} -\dfrac{\delta p}{\delta x} + f_{u,x} = \rho a_x \\ \\ -\dfrac{\delta p}{\delta y} + f_{u,y} = \rho a_y \\ \\ -\dfrac{\delta p}{\delta z} + f_{u,z} = \rho a_z \end{cases}$$

$$\xrightarrow{f_u = \text{zwaartekracht}} \quad -\frac{\delta p}{\delta z} - \rho g = 0$$

Bewegingsverg langs een stroomlijn

Wet van Bernoulli



$$\vec{F} = m\vec{a} = \rho\,dn\,dy\,dz.\vec{a} \begin{cases} a_s = \frac{dV}{dt} = \frac{dV}{ds}\frac{ds}{dt} = \frac{dV}{ds}.V \\ a_n = \frac{V^2}{R} \end{cases}$$

Fuitw. $\rightarrow$ stel enkel $\vec{G}$

$dG = \rho g\, dn\,dy\,dz$

$dG_s = -dG \sin\theta$

$dG_n = -dG \cos\theta$

s-richting

$$\sum \delta F_s = \rho\, ds\, dn\, dy\, a_s = \rho\, ds\, dn\, dy\; V \frac{dV}{ds}$$

$$= \delta G + \delta F_{P,s}$$

$$= -\rho g\, ds\, dn\, dy \sin\theta + \left(-\frac{\delta P}{\delta s}\, ds\, dn\, dy\right)$$

$$\Rightarrow \quad -\rho g \sin\theta - \frac{\delta P}{\delta s} = \rho V \frac{dV}{ds}$$

met $\sin\theta = \frac{dz}{ds}$

$V\frac{dV}{ds} = \frac{1}{2}\frac{d(V^2)}{ds}$

$dp = \frac{\delta P}{\delta s} ds + \frac{\delta P}{\delta n} \underbrace{dn}_{=0}$

want n=ct.

$$\Rightarrow \quad -\rho g \frac{dz}{ds} - \frac{dP}{ds} = \frac{\rho}{2}\frac{d(V^2)}{ds}$$

$$\Rightarrow \quad dp + \frac{1}{2}\rho\, d(V^2) + \rho g\, dz = 0$$

$$\Rightarrow \quad \int \frac{dP}{\rho} + \frac{1}{2}V^2 + gz = ct.$$

onsamendrukbaar

$$\Rightarrow \quad P + \frac{1}{2}\rho V^2 + \rho g z = ct.$$

geldig voor:
- ideale vl.
- permanente stroming
- langs stroomlijn

$$\Rightarrow \quad \frac{P}{\rho g} + \frac{V^2}{2g} + z = ct.$$

n-richting

$$\sum \delta F_n = \rho\, ds\, dn\, dy \cdot a_n = \rho\, ds\, dn\, dy\, \frac{V^2}{R}$$

$$= \delta G + \delta F_{p,n}$$

$$= -\rho g\, ds\, dn\, dy \cdot \cos\theta - \frac{\delta P}{\delta n}\, ds\, dn\, dy$$

$$= \left(-\rho g \cos\theta - \frac{\delta P}{\delta n}\right) ds\, dn\, dy \qquad \text{met } \cos\theta = \frac{dz}{dn}$$

$$\Rightarrow \quad \underbrace{-\rho g \frac{dz}{dn}}_{\text{gewicht} \perp \text{op stroomlijn}} \underbrace{- \frac{\delta P}{\delta n}}_{\text{drukgradiënt}} = \rho \frac{V^2}{R}$$

→ een grotere snelheid of dichtheid of een kleine kromtestraal vereisen een groter krachtenevenwicht om beweging te produceren.

↳ G verwaarl. → $\frac{\delta P}{\delta n} = -\rho \frac{V^2}{R}$

op tornado → partieel vacuüm

s = ct. ⊥ op stroomlijn → $\frac{dP}{dn} = \frac{\delta P}{\delta n}$

$\Rightarrow$

$$\int \frac{dP}{\rho} + \int \frac{V^2}{R} \cdot dn + gz = \text{cte.} \quad \text{(dwars op stroomlijn)}$$

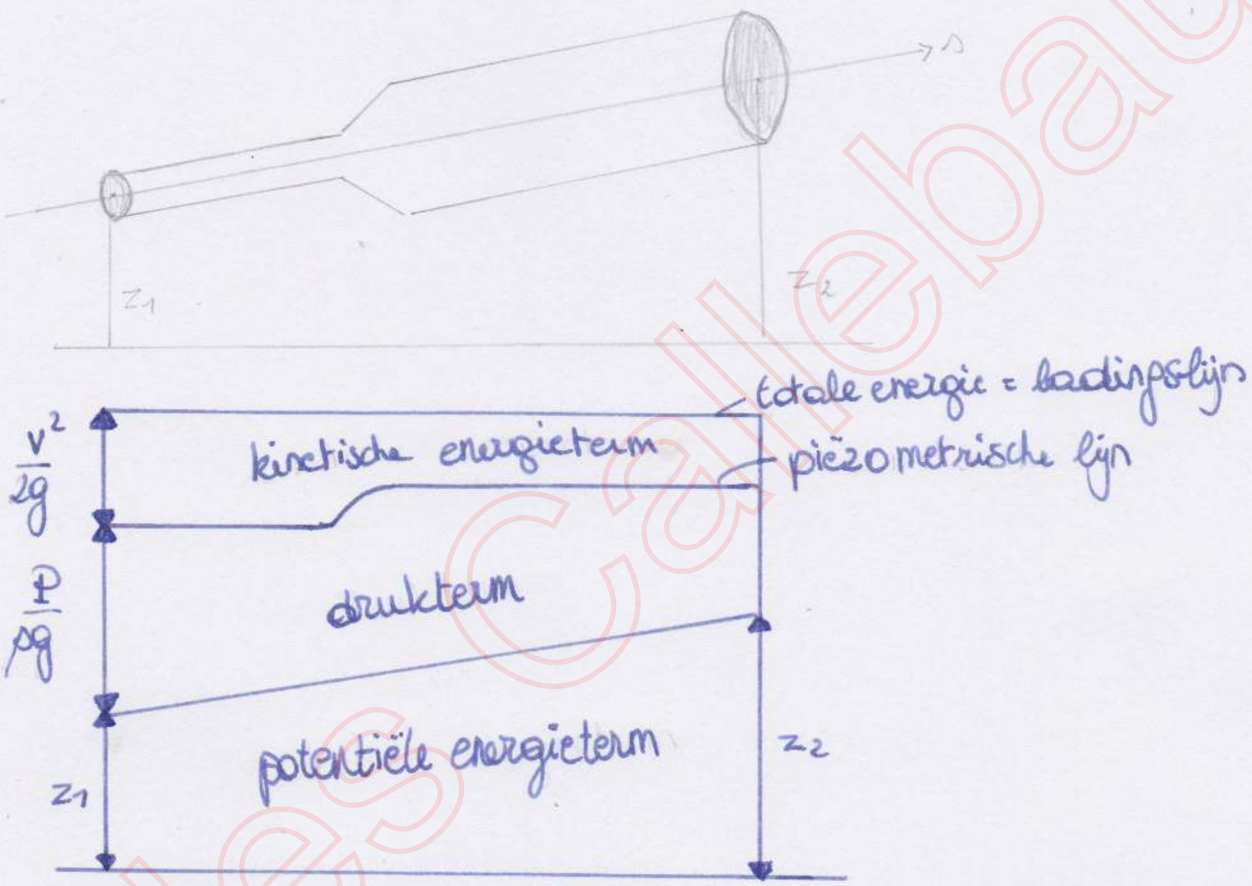
↓ onsamendrukbaar fluïdum

$$\frac{P}{\rho} + \int \frac{V^2}{R} dn + gz = \text{ct.}$$

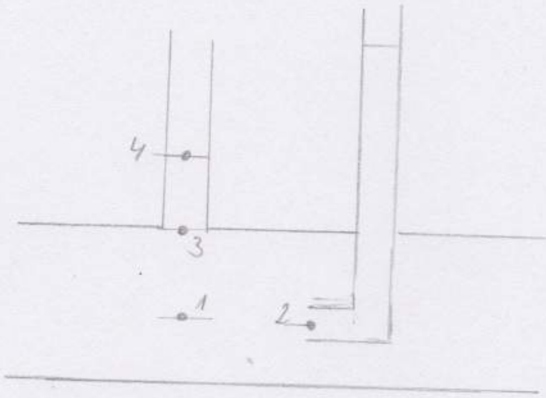
↓ rechtlijnig. $R = \infty$

$$\Longrightarrow \quad z + \frac{P}{\rho g} = \text{ct.}$$

ladingslijn



Statische & Dynamische druk



Statische druk: druk = vloeistof in rust

① $P_1 = P_3 + \rho g h_{31}$

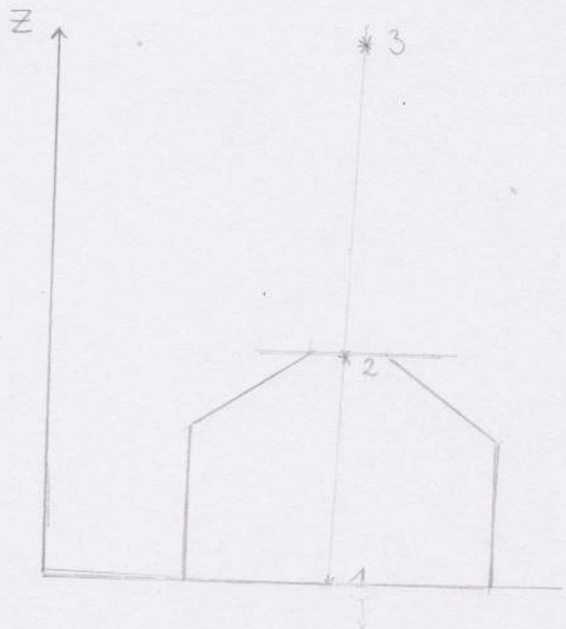
Dynamische druk: tweede term v. Bernoulli, $\frac{1}{2}\rho v^2$

② $P_2 = P_1 + \frac{1}{2}\rho V_1^2$

$v_2 = 0$

Toepassingen op de bewegingsverg.

Uitstromen van water uit een fontein



$\rho = 1000\ kg/m^3$

$P_1 = 5$ bara → absolute druk

$z_2 = 0,1\ m$

$$z + \frac{P}{\rho g} + \frac{v^2}{2g} = ct. \quad \text{op stroomlijn}$$

$$z + \frac{P}{\rho g} = ct. \quad \text{dwars op stroomlijn}$$

① $\dfrac{5.10^5}{1000.10} = cte.$

② $0,1 + \dfrac{P_{atm}}{\rho g} + \dfrac{V_2^2}{2g} = cte.$

dwars: $\dfrac{P_{atm}}{\rho g} = \dfrac{P_2}{\rho g}$

③ $H + \dfrac{P_{atm}}{\rho g} + \dfrac{V_3^2}{2g} = cte.$

Castelli:

$A_1 V_1 = A_2 V_2$

$A_1 >> A_2 \rightarrow V_1 << V_2$

↑ te verwaarl.

① = ② ⇒ $V_2 = 28,25\ m/s$

② = ③ ⇒ $\underline{\underline{H = 40m}}$

Uitstroming uit een vat

① $H + \frac{P_{atm}}{\rho g} = ct.$

② $\frac{P_{atm}}{\rho g} + \frac{v_2^2}{2g} = ct.$

v_1 verwaarl. door Castelli

$A_1 v_1 = A_2 v_2$

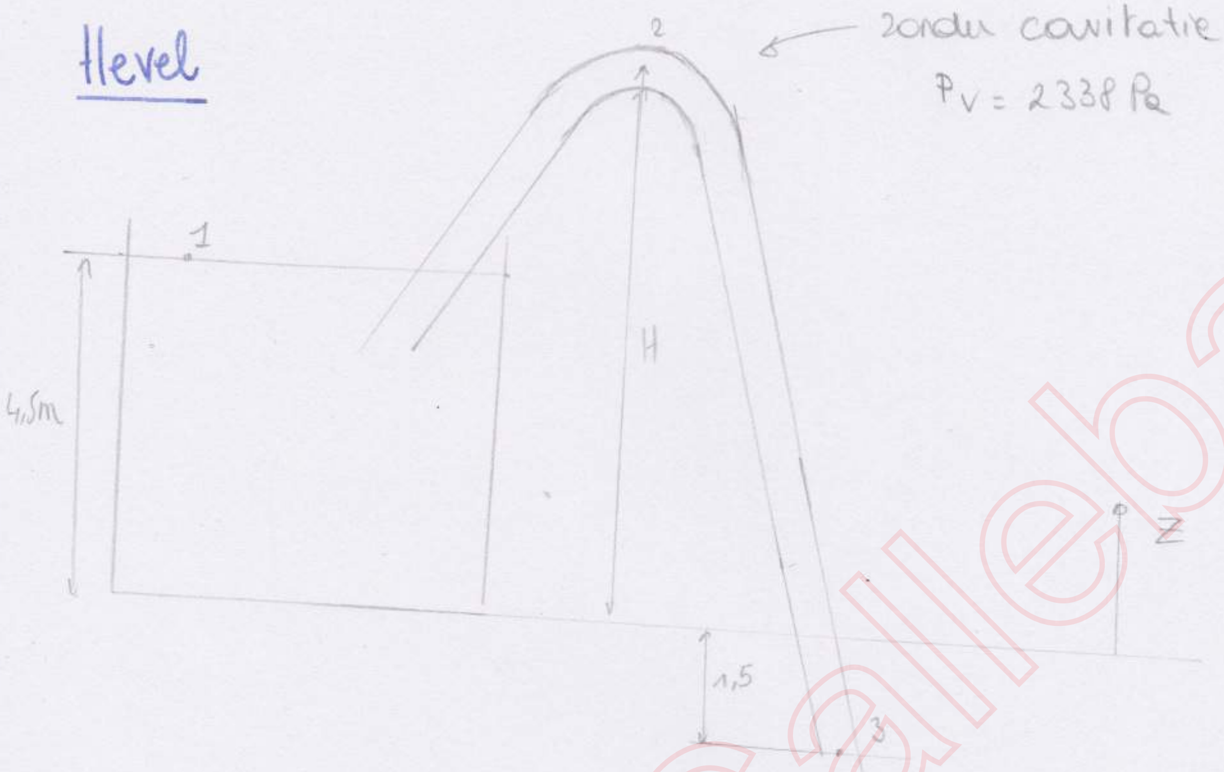
$A_1 >> A_2$

① = ② $\Rightarrow$ $v_2 = \sqrt{2gH}$

↳ gemiddelde snelheid

debiet: $Q = v_2 \cdot A = \frac{\pi d^2}{4}\sqrt{2gH}$

Hevel



① $4{,}5m + \frac{P_{atm}}{\rho g} = ct$

② $H + \frac{P_2}{\rho g} + \frac{v_2^2}{2g} = ct.$

③ $-1{,}5m + \frac{P_{atm}}{\rho g} + \frac{v_3^2}{2g} = ct.$

① = ③ $\Rightarrow v_3 = 10{,}85 \; m/s$

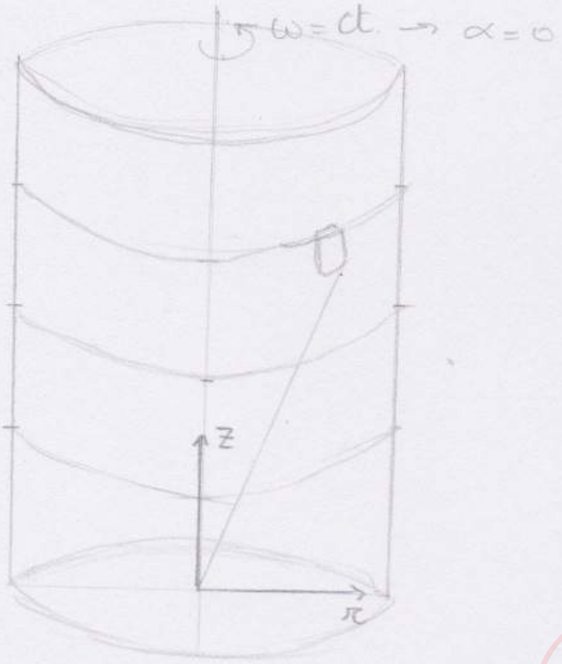
① = ② $\Rightarrow \frac{P_2}{\rho g} = (4{,}5 - H) - \frac{v_2^2}{2g} + \frac{P_{atm}}{\rho g}$

We weten $v_2 = v_3$ (castelli)

$$H = -\frac{v_2^2}{2g} + \frac{P_{atm}}{\rho g} - \frac{P_2}{\rho g} + 4{,}5$$

$\underline{\underline{H = 8{,}59 \, m}}$

vloeistof onderworpen aan een rotatie (vert. as)



$$\begin{cases} v_r = 0 \\ v_z = 0 \\ v_\theta = r\omega \end{cases} \rightarrow \begin{cases} a_r = -r\omega^2 \\ a_z = 0 \\ a_\theta = 0 \end{cases}$$

$$\Downarrow$$

$$\begin{cases} \dfrac{\delta P}{\delta r} = -\rho a_r \\ \dfrac{\delta P}{\delta z} = -\rho g \\ \dfrac{\delta P}{\delta \theta} = 0 \end{cases}$$

$$dP = \frac{\delta P}{\delta r} dr + \frac{\delta P}{\delta \theta} d\theta + \frac{\delta P}{\delta z} dz$$

$$dP = \rho r \omega^2 dr - \rho g \, dz$$

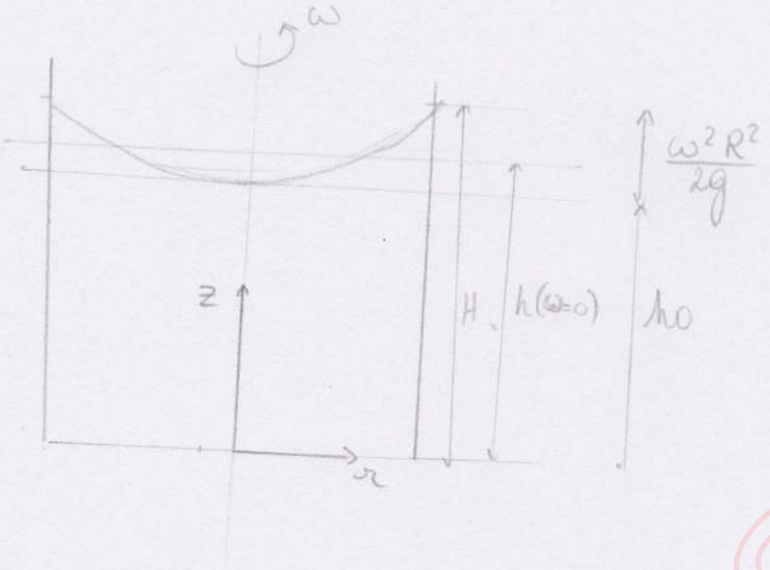
$$\Delta P = \int \rho r \omega^2 dr - \rho g \Delta z$$

$$\Delta P = \frac{\rho \omega^2}{2} \Delta r^2 - \rho g \Delta z$$

$$\downarrow \quad P = ct.$$

$$z = \frac{\omega^2 r^2}{2g} + cte.$$

Bepalen integratieconstante



Vat te klein (H = hoogte vat)

$$z = H$$

$$H = h_0 + \frac{\omega^2 r^2}{2g} \rightarrow h_0 = H - \frac{\omega^2 R^2}{2g}$$

Vat groot genoeg

$$V_{begin} = V_{eind}$$

$$\pi R^2 h = \int_0^R \underbrace{2\pi r\,dr}_{opp.} \cdot \underbrace{z}_{hoogte} = \int_0^R 2\pi r\,dr \cdot \left(h_0 + \frac{\omega^2 r^2}{2g}\right)$$

$$= h_0 \pi R^2 + \frac{\omega^2 R^4}{4g}\pi$$

$$\Rightarrow \pi R^2 h = h_0 \pi R^2 + \frac{\omega^2 R^4}{4g}\pi$$

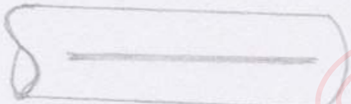
$$\Rightarrow h = h_0 + \frac{\omega^2 R^2}{4g} \Rightarrow H = h_0 + \frac{\omega^2 r^2}{2g} \Rightarrow H = h - \frac{\omega^2 R^2}{4g} + \frac{\omega^2 r^2}{2g}$$

H4: Beweging van werkelijke fluïda

Stroming van werkelijke fluïda

↳ wrijvingsverliezen ← schuifspanningen
   ↳ snelheidsprofiel

Laminaire ∝ turbulente stroming



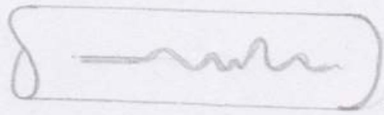
lage snelheden

hoge viscositeit

goed geleide stroming

$u_a = \bar{U}$

$u_s = \bar{U}$



hogere snelheden

axiale snelheidscomp.:

$u_a = \bar{U}_a + \underbrace{U'_a}_{\text{turbulente fluctuatie}}$

Reynoldsgetal

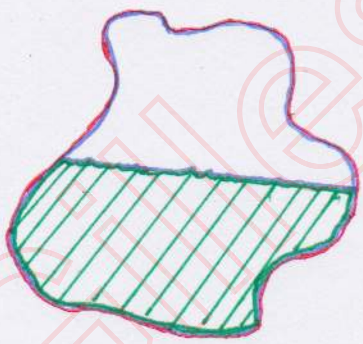
$$Re = \frac{\rho D V}{\mu} = \frac{DV}{\nu} \quad \begin{matrix} \mapsto \text{zwaartekracht} \\ \mapsto \text{viscositeitskracht} \end{matrix}$$

$$\begin{cases} \rho : \text{dens. } [kg/m^3] \\ D : \text{diam. } [m] \\ V : \text{gem. snelheid } [m/s] \\ \mu : \text{dyn. viscositeit } [Pa.s] \end{cases}$$

$Re < 2000 \rightarrow$ laminair

$Re > 4000 \rightarrow$ turbulent

Hydraulische diameter v.e leiding

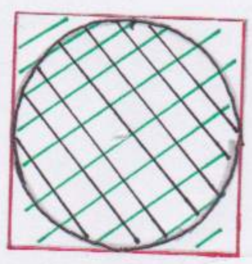


S_n = natte sectie

P_N = natte omtrek

$$\underbrace{d_h}_{\text{hydr. diameter}} = 4 \frac{S_n}{P_n}$$

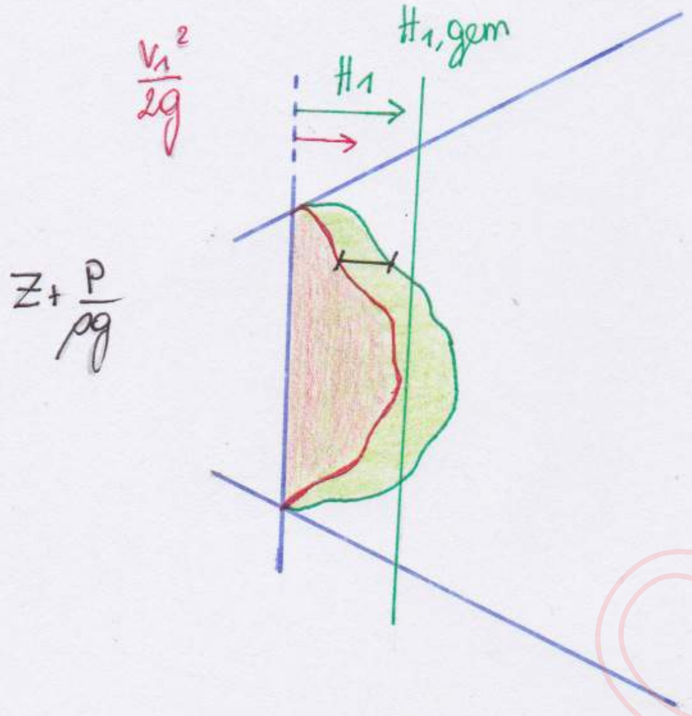
vb.



$$d_h = 4 \frac{z^2}{4z} = z$$

Bepalen α

↳ verliesterm



Bernoulli

(A₁) $Z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} = H_1 \rightarrow H_{1,gem} = Z_1 + \frac{P_1}{\rho g} + \alpha \frac{v_{1,gem}^2}{2g}$

Vermogen

(dA₁) $d\mathcal{P} = \rho g H_1 . dQ_1$

$= \rho g \left(Z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g} \right) (dA_1 . v_1)$

(A₁) $P = \rho g H_{1,gem} Q_1 \hookrightarrow H_{1,gem} = \int_{A_1} \frac{\left(Z_1 + \frac{P_1}{\rho g} + \frac{v_1^2}{2g}\right) v_1 \, dA}{Q_1}$

$$H_{1,gem} = \frac{Z_1 + \frac{P_1}{\rho g}}{Q_1}\int_{A_1} V_1 \, dA_1 + \frac{1}{2gQ_1}\int_{A_1} V_1^3 \, dA_1$$

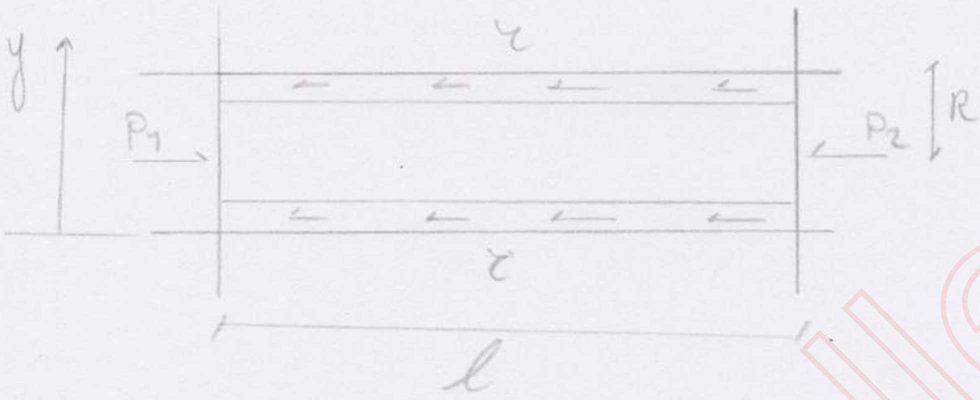
$$= Z_1 + \frac{P_1}{\rho g} + \underbrace{\frac{1}{2gQ_1}\int_{A_1} V_1^3 \, dA_1}_{\text{schrijven als}}$$

$$\alpha_1 \cdot \frac{V_{1,gem}^2}{2g}$$

$$\Rightarrow \int_{A_1} \frac{V_1^3 \, dA_1}{2gQ_1} \frac{V_{1,gem}^2}{V_{1,gem}^2} = \underbrace{\int_{A_1} \frac{V_1^3 \, dA_1}{Q_1 \cdot V_{1,gem}^2}}_{= \alpha} \cdot \frac{V_{1,gem}^2}{2g}$$

$$\alpha = \frac{\int_{A_1} V_1^3 \, dA_1}{V_{1,gem}^3 A_1}$$

Cirkelvormige doorsnede



Bernoulli

$$y_1 + \frac{P_1}{\rho g} + \alpha_1 \frac{V_{1,gem}^2}{2g} = y_2 + \frac{P_2}{\rho g} + \alpha_2 \frac{V_{2,gem}^2}{2g} + h_w$$

↳ hor. : $y_1 = y_2$

Cast. : $A_1 = A_2$, $V_1(r) = V_2(r) \rightarrow \alpha_1 = \alpha_2$

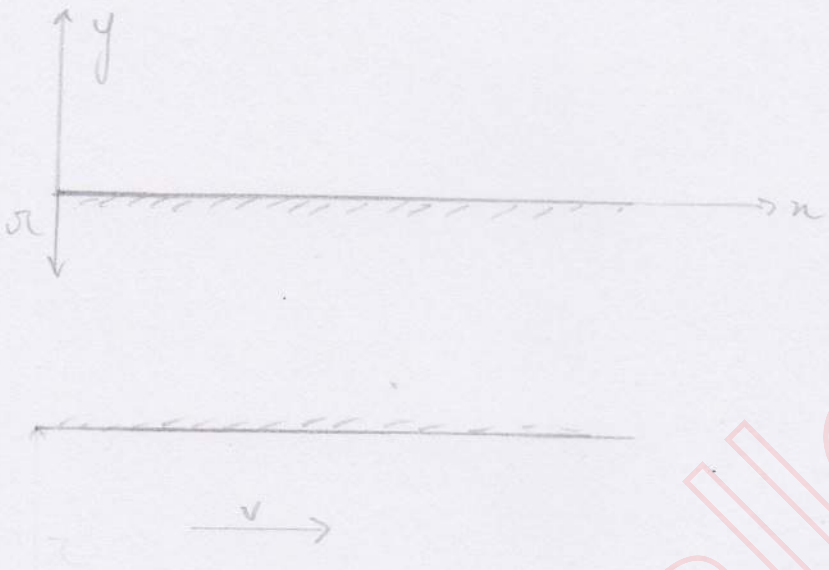
$$\Rightarrow \frac{P_1 - P_2}{\rho g} = h_w = \frac{P_w}{\rho g}$$

Mechanisch evenwicht

$$P_1 \pi r^2 = P_2 \pi r^2 + \tau (2\pi r l)$$

$$\tau = (P_1 - P_2) \cdot \frac{r}{2l} \Rightarrow \tau = P_w \left(\frac{r}{2l}\right)$$

Vloeistof (veronderstel Newt.)



$$\tau = \mu \frac{du}{\delta y}$$

$$\tau = \mu \frac{\delta v}{(-\delta r)} \xrightarrow[\text{symm.}]{\text{cil.}} \tau = -\mu \frac{dv}{dr}$$

$$\Rightarrow \int_v^0 dv = -\int_r^R \left(\frac{P_w}{2l} \cdot r\right) \frac{dr}{\mu}$$

$$\Rightarrow \quad \boxed{v = \frac{P_w}{4\mu l}\left(R^2 - r^2\right)}$$

Debiet:

$$Q = \int_A v \, dA = \int_0^R v(r) \, 2\pi r \, dr$$

$$= \int_0^R \frac{P_w (R^2 - r^2)}{4\mu\ell} \, 2\pi r \, dr$$

$$Q = \frac{P_w}{8\mu\ell} \pi R^4$$

Snelheid

$$v_{gem} = \frac{Q}{A} = \frac{P_w}{8\mu\ell} R^2 = \frac{v_{max}}{2}$$

α

$$\alpha = \frac{\int_A v^3 \, dA}{v_{gem}^3 \, A} = 8 \int \left(1 - \frac{r^2}{R^2}\right) \cdot \frac{1}{R^2} \, 2r \, dr$$

$$u = \left(1 - \left(\frac{r}{R}\right)^2\right)$$

$$du = \frac{-2r \, dr}{R^2}$$

$$= -8 \int_1^0 u^3 \, du$$

$r = R \downarrow u = 0$ $\quad$ $r = 0 \downarrow u = 1$

$$\alpha = 2$$

wrijvingscoëff.

$$h_w = \lambda \frac{l}{2R} \cdot \frac{V_{,gem}^2}{2g}$$

$$h_w = \frac{P_w}{\rho g}$$

$$P_w = \frac{8 \nu l V_{,gem}}{R^2}$$

$$\left(V_{gem} = \frac{P_w R^2}{8 \nu l} \right)$$

$$\sum_1^2 \Delta F = \underbrace{\xi_i}_{\text{verliesfactor} \sim l} \frac{v_i^2{,gem}}{2g}$$

$$\xi_i = \lambda \frac{l}{D}$$

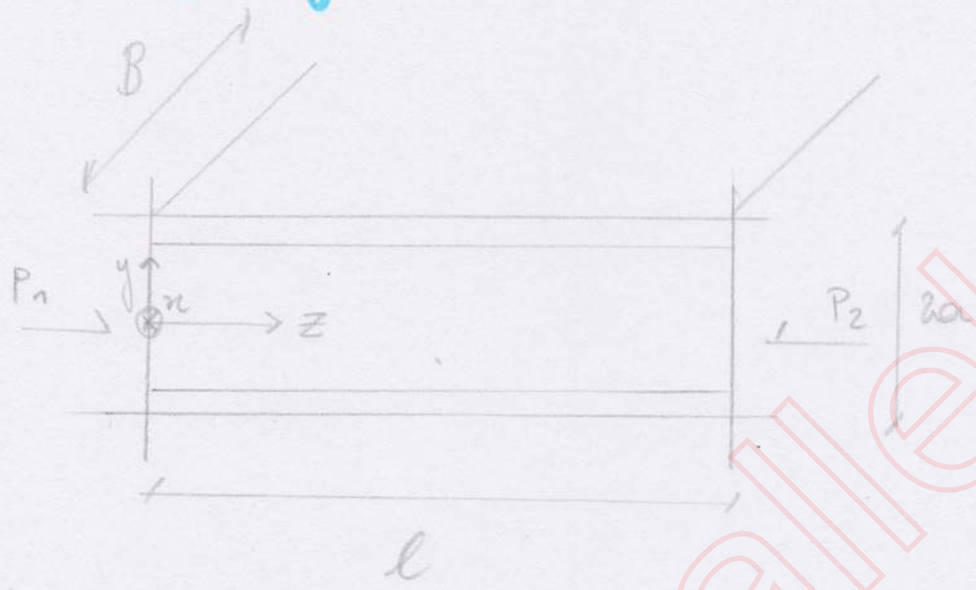
$$\frac{8 \nu l \bar{v}}{R^2} \frac{1}{\rho g} = \lambda \frac{l}{2R} \frac{\bar{v}^2}{2g}$$

$$\lambda = \frac{2 \nu 8 . 2}{R \rho \bar{v}} \quad \text{met } \nu = \frac{\nu}{\rho}$$

$$\lambda = \frac{64}{2R} \frac{\nu}{\bar{v}}$$

$$\lambda = 64 \frac{\nu}{D \bar{v}} \Rightarrow \lambda = \frac{64}{Re}$$

Evenwijdige platen



Bernoulli

idem cirk. $\qquad h_w = \frac{P_1 - P_2}{\rho g} = \frac{P_w}{\rho g}$

Mech. ev.

$$P_1\,(2y).B = P_2\,(2y).B + \tau . B\ell 2$$

$$\tau = P_w \frac{y}{\ell}$$

vloeistof

$$\tau = -\mu \frac{dv}{dy} = P_w \frac{y}{\ell} \Rightarrow \int_v^0 dv = \int_y^a - P_w \frac{y}{\ell} \cdot \frac{dy}{\mu}$$

$$v = \frac{P_w}{2\mu\ell}\left(a^2 - y^2\right)$$

Debiet

$$Q = \int_A v\, dA = \int_0^a v\, 2B\, dy = \frac{P_w}{\mu l} B \int_0^a (a^2 - y^2)\, dy$$

$$Q = \frac{P_w}{3\mu l} B a^3$$

α

$$\alpha = \frac{54}{35}$$

$\bar{V}$

$$\bar{V} = \frac{Q}{A} = \frac{P_w a^2}{3\mu l} \left(= \frac{2}{3} v_{max} \right)$$

λ

$$h_w = \lambda \frac{l}{2a} \frac{v^2_{gem}}{2g}$$

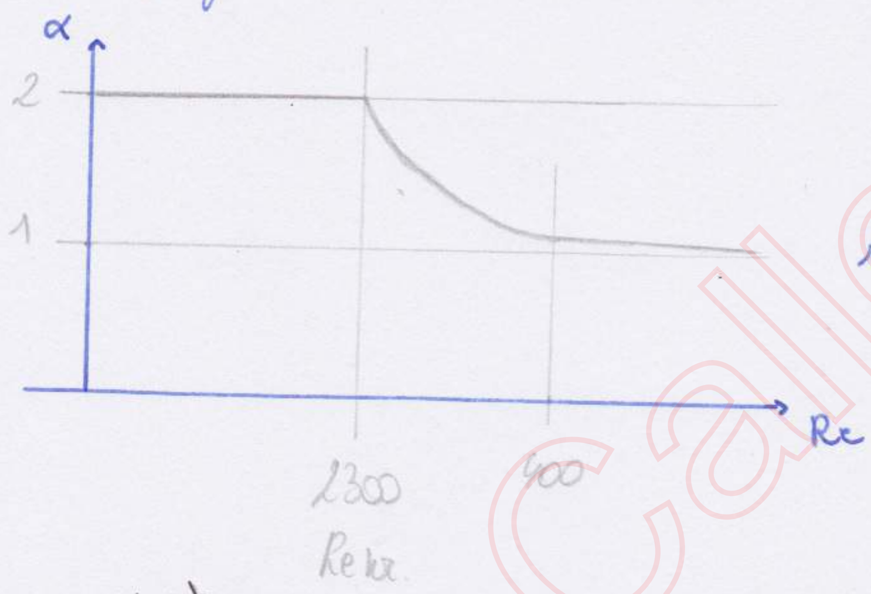
$$\parallel$$

$$\frac{P_w}{\rho g} = \frac{\bar{v}\, 3\mu l}{a^2} \cdot \frac{1}{\rho g}$$

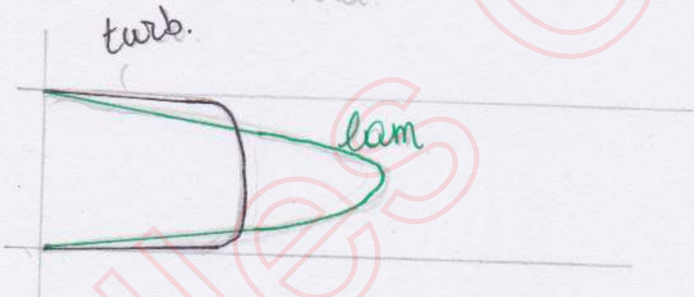
$$\longrightarrow \lambda = \frac{24}{Re}$$

turbulente stroming

Snelheidsprofiel



snel $\alpha = 1$

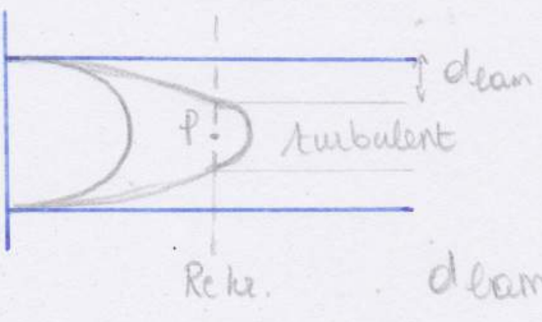


laminaire sublaag

$$Re \rightarrow \bar{Re} = \frac{\bar{v} D}{\nu} \rightarrow \begin{cases} < Re_{kr} = \text{lam} \\ > Re_{kr} = \text{turb.} \end{cases}$$

$$Re \rightarrow Re = \frac{v D}{\nu}$$

start: $v=0$ → laminair (parabool)



P: start turb.

lam. stroming → veel stiler

$d_{lam} \approx \frac{1}{Re}$ = laminaire sublaag

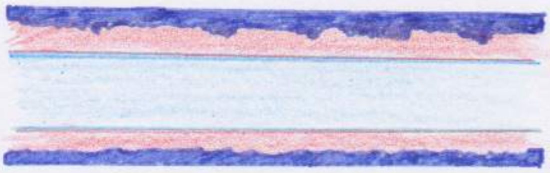
Opp. ruwheid

↳ k = ruwheid

Hydraulisch glad

Re klein beetje > Re_{kr}.

↳ beperkte turb. str.



$\updownarrow \delta_{lam}$ → gladde buis

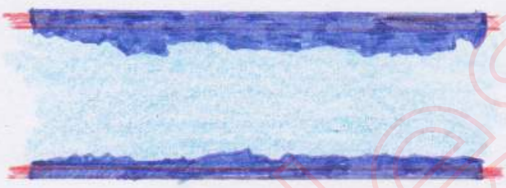
$\updownarrow k$

$$\lambda = f(Re)$$

$$\frac{Re \cdot k}{D} < 23$$

Hydraulisch ruw

$Re >> Re_{kr}$.

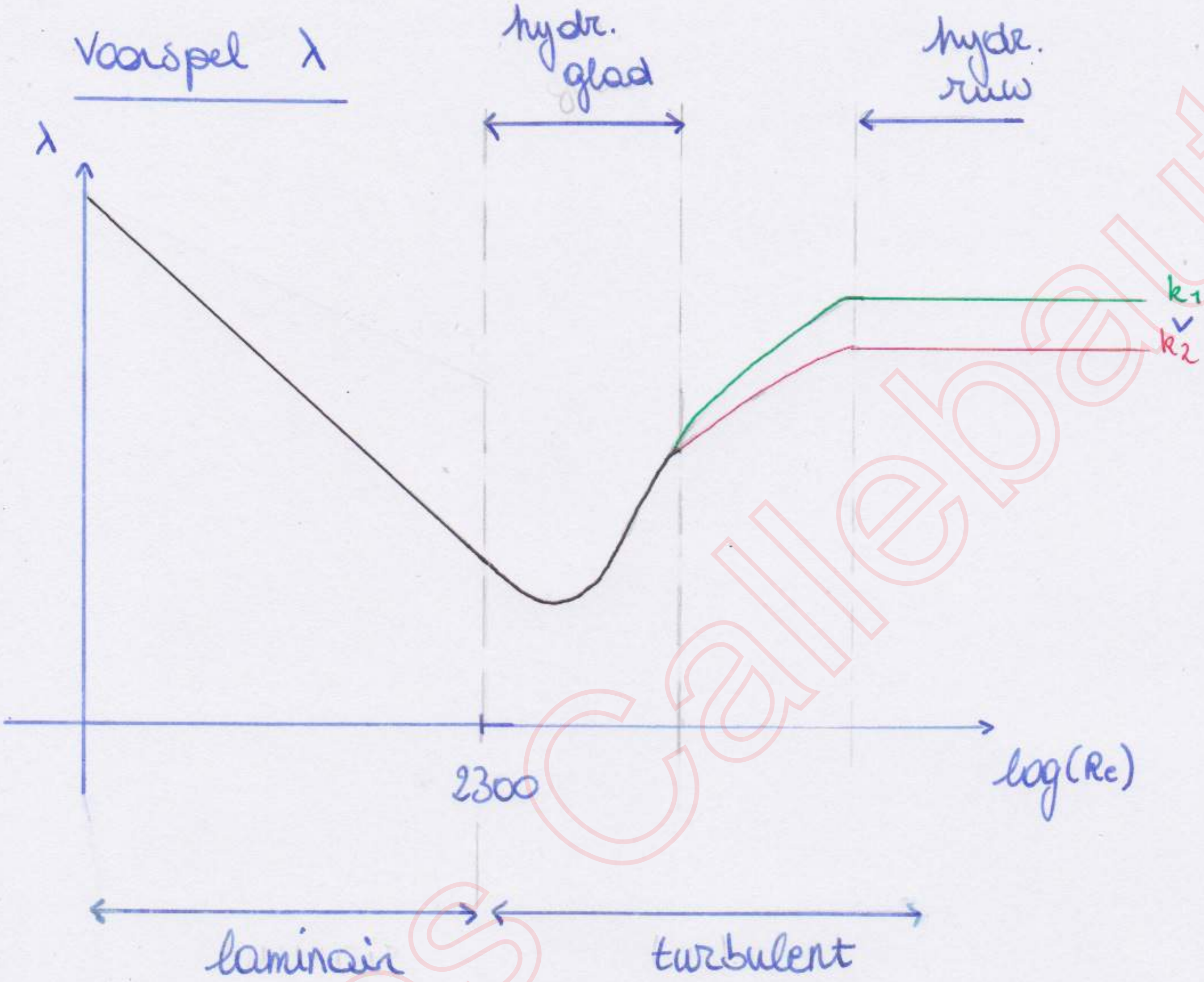


$$\lambda = f(k) \qquad \frac{Re \cdot k}{D} > 560$$

Overgang:

$$\lambda = f(Re, k) \qquad 23 < \frac{Re \cdot k}{D} < 560$$

Voorspel λ



$\lambda = \frac{64}{Re}$

4-23
4-24

gemiddelde

4-25

(p 4-17)

Drukval in leidingcomponenten

→ Bernoulli → $z_1 + \frac{p_1}{\rho g} + \alpha_1 \frac{v_1^2}{2g} = z_2 + \frac{p_2}{\rho g} + \alpha_2 \frac{v_2^2}{2g} + \sum_1^2 h_{verlies}$

met $$\sum_1^2 h_{verlies} = \underbrace{\sum_1^2 h_w}_{= \frac{\lambda l}{D} \cdot \frac{v^2}{2g}} + \underbrace{\sum_1^2 h_{plaatselijk}}_{= \zeta_i \cdot \frac{v_i^2}{2g}}$$

→ altijd grootste snelheid → max in rekening brengen van verliezen.

→ uitstromen in vat → $\zeta_i = 1$ (turbulentie)

→ Stel hydraulisch ruw & turbulent

$$\lambda = \frac{1}{\left[2 \log\left(\frac{D}{k}\right) + 1{,}138\right]^2}$$

→ bochten: $\zeta_i = a \cdot b \cdot c$ → af te lezen in grafiek

→ controle: hydr. ruw: $\frac{Re \cdot k}{D} > 560$ met $Re = \frac{v \cdot D}{\nu}$

turbulent: $Re > 2300$

Leidingsystemen

Serie

↳ $Q_1 = Q_2 = \dots$

$h_w = h_{w,1} + h_{w,2} + \dots$

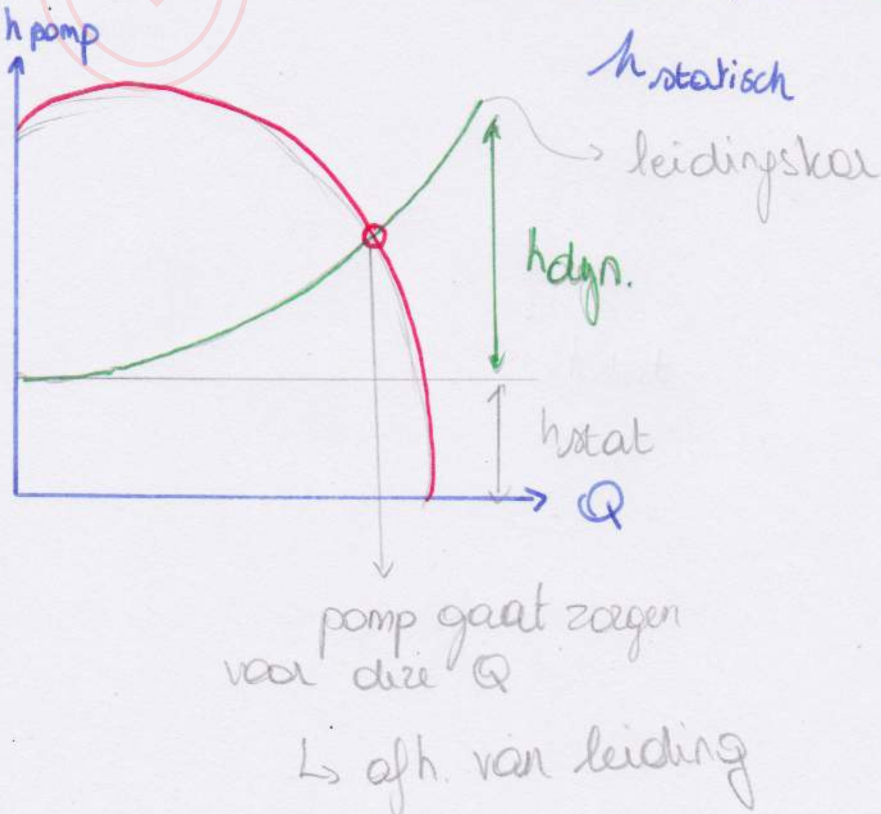
Parallel

↳ $Q = Q_1 + Q_2 + \dots$

$h_{w_1} = h_{w_2} = \dots$

Pomp

$$h_{pomp} = \underbrace{(Z_2 - Z_1) + \frac{(P_2 - P_1)}{\rho g} + \frac{(\alpha_2 V_2^2 - \alpha_1 V_1^2)}{2g}}_{h_{statisch}} + \underbrace{\sum_1^2 h_{verlies}}_{h_{dynamisch}}$$



$h_{dynamisch}$
- h_{pl} $\sim Q^2$
- h_w
  - lam. $\frac{\lambda l}{D} \underbrace{\frac{v^2}{2g}}_{\sim Q^2}$, $\lambda \sim \frac{1}{Q}$
  - turb.
    - glad: $h_w \sim Q^k$
    - ruw: $h_w \sim Q^2$

⟶ $h_w \sim Q^2$ (meestal)